

TEMA 2.- GEOMETRÍA DEL ELIPSOIDE DE REVOLUCIÓN

EJERCICIO 2.1

$$a = 5; \quad c = 3 \qquad a^2 = b^2 + c^2 \qquad \Rightarrow \qquad b = \sqrt{a^2 - c^2} = \sqrt{25 - 9} = 4$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \Rightarrow \qquad \boxed{\frac{x^2}{25} + \frac{y^2}{16} = 1}$$

EJERCICIO 2.2

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} + \frac{z^2}{b^2} = 1 \qquad \Rightarrow \qquad \boxed{\frac{x^2}{25} + \frac{y^2}{25} + \frac{z^2}{16} = 1}$$

$$f = \frac{a-b}{a} = \left\{ \begin{array}{l} a=5 \\ b=4 \end{array} \right\} = \frac{5-4}{5} \qquad \Rightarrow \qquad \boxed{f = 1/5 = 0,2}$$

$$e = \frac{c}{a} = \frac{3}{5} \qquad \Rightarrow \qquad \boxed{e = 3/5 = 0,6}; \qquad e' = \frac{c}{b} = \frac{3}{4} \qquad \Rightarrow \qquad \boxed{e' = 3/4 = 0,75}$$

EJERCICIO 2.3

$$f = 1 - \frac{b}{a} \qquad \Rightarrow \qquad \boxed{b = a \cdot (1 - f)}$$

$$c = \sqrt{a^2 - b^2} = \sqrt{a^2 - a^2(1-f)^2} = a\sqrt{1 - (1-f)^2} = a\sqrt{1 - 1 + 2f - f^2} \qquad \Rightarrow \qquad \boxed{c = a\sqrt{f(2-f)}}$$

$$e = \frac{c}{a} = \frac{a\sqrt{f(2-f)}}{a} \qquad \Rightarrow \qquad \boxed{e = \sqrt{f(2-f)}}; \qquad e' = \frac{c}{b} = \frac{a\sqrt{f(2-f)}}{a(1-f)} \qquad \Rightarrow \qquad \boxed{e' = \frac{\sqrt{f(2-f)}}{1-f}}$$

$$\left. \begin{array}{l} a = 6.378.388 \text{ m} \\ f = 1/297 \end{array} \right\} \qquad \begin{array}{l} \boxed{b = 6.356.911,95 \text{ m}} \\ \boxed{e = 0,08199188998} \end{array} \qquad \begin{array}{l} \boxed{c = 522.976,09 \text{ m}} \\ \boxed{e' = 0,08226888961} \end{array}$$

EJERCICIO 2.4

$$\left. \begin{array}{l} a = 6.378.137 \text{ m} \\ f = 1/298,257 \end{array} \right\} \qquad \begin{array}{l} b = a(1-f) \qquad \Rightarrow \qquad \boxed{b = 6.356.752,30 \text{ m}} \\ c = a\sqrt{f(2-f)} \qquad \Rightarrow \qquad \boxed{c = 521.854,20 \text{ m}} \end{array}$$

$$e = \sqrt{f(2-f)} \Rightarrow \boxed{e = 0,08181922146}$$

$$e' = \frac{\sqrt{f(2-f)}}{1-f} \Rightarrow \boxed{e' = 0,08209446887}$$

EJERCICIO 2.5

$$N = \frac{a}{w} = \frac{a}{\sqrt{1-e^2 \operatorname{sen}^2 \varphi}} \Rightarrow \boxed{N = \frac{a}{\sqrt{1-f(2-f) \operatorname{sen}^2 \varphi}}}$$

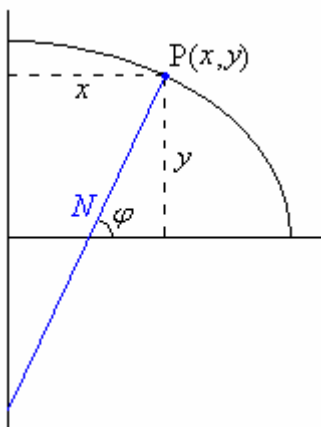
Internacional.-

$$N = \frac{a}{\sqrt{1-f(2-f) \operatorname{sen}^2 \varphi}} = \frac{6.378.388 \text{ m}}{\sqrt{1-\frac{1}{297} \left(2-\frac{1}{297}\right) \operatorname{sen}^2 30^\circ}} = \frac{6.378.388 \text{ m}}{0,999159312} \Rightarrow \boxed{N = 6.383.754,74 \text{ m}}$$

WGS84.-

$$N = \frac{a}{\sqrt{1-f(2-f) \operatorname{sen}^2 \varphi}} = \frac{6.378.137 \text{ m}}{\sqrt{1-\frac{1}{298,257} \left(2-\frac{1}{298,257}\right) \operatorname{sen}^2 30^\circ}} = \frac{6.378.137 \text{ m}}{0,999162851} \Rightarrow \boxed{N = 6.383.480,92 \text{ m}}$$

EJERCICIO 2.6



$$\left. \begin{aligned} x &= N \cos \varphi \\ y &= N (1-e^2) \operatorname{sen} \varphi \end{aligned} \right\} \begin{array}{l} \text{Hayford} \\ a = 6.378.388 \text{ m} \\ f = 1/297 \end{array} \quad \varphi = 30^\circ$$

$$w = \sqrt{1-f(2-f) \operatorname{sen}^2 \varphi} \Rightarrow w = 0,999159312$$

$$N = \frac{a}{w} = \frac{6.378.388 \text{ m}}{0,999159312} \Rightarrow N = 6.383.754,74 \text{ m}$$

$$x = N \cos \varphi = 6.378.754,74 \cos 30^\circ \Rightarrow \boxed{x = 5.528493,78 \text{ m}}$$

$$y = N (1-e^2) \operatorname{sen} \varphi = N (1-f(2-f)) \operatorname{sen} \varphi = 6.378.754,74 \left(1-\frac{1}{297} \left(2-\frac{1}{297}\right)\right) \operatorname{sen} 30^\circ \Rightarrow \boxed{y = 3.170.419,43 \text{ m}}$$

EJERCICIO 2.7

$$\rho = \frac{a(1-e^2)}{w^3} = \frac{a(1-f(2-f))}{(1-e^2 \text{sen}^2 \varphi)^{3/2}} \Rightarrow \boxed{\rho = \frac{a(1-f(2-f))}{(1-e^2 \text{sen}^2 \varphi)^{3/2}}} \quad \boxed{N = \frac{a}{\sqrt{1-f(2-f) \text{sen}^2 \varphi}}}$$

$$\left. \begin{array}{l} a = 6.378.137 \text{ m} \\ f = 1/298,257 \end{array} \right\} \quad \boxed{\rho = 6.351.377,08 \text{ m}} \quad \boxed{N = 6.383.480,92 \text{ m}}$$

EJERCICIO 2.8

$$R_\alpha = \frac{N \rho}{\rho \text{sen}^2 \alpha + N \text{cos}^2 \alpha}$$

$$\rho = \frac{a(1-f(2-f))}{(1-e^2 \text{sen}^2 \varphi)^{3/2}} = \frac{6.335.508,202 \text{ m}}{0,9949622369} \Rightarrow \boxed{\rho = 6.367.586,60 \text{ m}}$$

$$N = \frac{a}{\sqrt{1-f(2-f) \text{sen}^2 \varphi}} = \frac{6.378.388 \text{ m}}{0,9983179178} \Rightarrow \boxed{N = 6.389.135,05 \text{ m}}$$

$$R_\alpha = \frac{N \rho}{\rho \text{sen}^2 \alpha + N \text{cos}^2 \alpha} \Rightarrow R_{20} = \frac{N \rho}{\rho \text{sen}^2 20^\circ + N \text{cos}^2 20^\circ} \Rightarrow \boxed{R_{20} = 6.370.099,78 \text{ m}}$$

En efecto es: $\boxed{\rho < R_\alpha < N}$

EJERCICIO 2.9

$$w = \sqrt{1-f(2-f) \text{sen}^2 \varphi} \Rightarrow w = 0,998500217 \Rightarrow w^3 = 0,995507395$$

$$\rho = \frac{a(1-e^2)}{w^3} = \frac{a(1-f(2-f))}{w^3} = \frac{6.335.439,295 \text{ m}}{0,995507395} \Rightarrow \rho = 6.364.030,37 \text{ m}$$

$$N = \frac{a}{w} = \frac{6.378.137 \text{ m}}{0,998500217} \Rightarrow N = 6.387.717,19 \text{ m}$$

$$R = \sqrt{N \cdot \rho} \Rightarrow \boxed{R = 6.375.862,78 \text{ m}}$$

EJERCICIO 2.10

$$\left. \begin{array}{l} a = 6.378.388 \text{ m} \\ f = 1/297 \end{array} \right\} \quad w = \sqrt{1-f(2-f) \text{sen}^2 \varphi} \Rightarrow \left\{ \begin{array}{l} w_{36} = 0,998838012 \\ w_{43} = 0,998435345 \end{array} \right.$$

$$N_{36} = \frac{a}{w_{36}} = \frac{6.378.388 \text{ m}}{0,998838012} = 6.385.808,233 \text{ m}; \quad N_{43} = \frac{a}{w_{43}} = \frac{6.378.388 \text{ m}}{0,998435345} = 6.388.383,61 \text{ m}$$

$$S_p = N \cos \varphi \Delta \lambda$$

$$S_{p36} = N_{36} (\cos 36^\circ) \frac{\pi}{3600 \cdot 180} \Rightarrow \boxed{S_{p36} = 25,05 \text{ m}}$$

$$S_{p43} = N_{43} (\cos 43^\circ) \frac{\pi}{3600 \cdot 180} \Rightarrow \boxed{S_{p43} = 22,65 \text{ m}}$$

EJERCICIO 2.11

$$S_m = \rho_m \Delta \varphi \left[1 + \frac{e^2}{8} \Delta \varphi^2 \cos 2\varphi_M \right]$$

$$\left. \begin{array}{l} a = 6.378.388 \text{ m} \\ f = 1/297 \end{array} \right\} \Rightarrow \begin{array}{l} e^2 = f(2-f) \\ e^2 = 0,006722670022 \end{array}$$

$$w^3 = (1 - e^2 \operatorname{sen}^2 \varphi)^{3/2} \Rightarrow \begin{cases} w_{36}^3 = 0,996518085 \\ w_{43}^3 = 0,995313378 \end{cases}$$

$$\rho_m = \frac{a(1-e^2)}{w^3} = \frac{a(1-f(2-f))}{w^3} \Rightarrow \begin{cases} \rho_{m36} = 6.357.644,98 \text{ m} \\ \rho_{m43} = 6.365.340,15 \text{ m} \end{cases}$$

$$S_{m36} = \rho_{m36} \Delta \varphi \left[1 + \frac{e^2}{8} \Delta \varphi^2 \cos 2\varphi_M \right] = 6.357.644,98 \cdot \frac{\pi}{3600 \cdot 180} \left[1 + \frac{0,006722670022}{8} \left(\frac{\pi}{3600 \cdot 180} \right)^2 \cos 72^\circ \right]$$

$$S_{m43} = \rho_{m43} \Delta \varphi \left[1 + \frac{e^2}{8} \Delta \varphi^2 \cos 2\varphi_M \right] = 6.365.340,15 \cdot \frac{\pi}{3600 \cdot 180} \left[1 + \frac{0,006722670022}{8} \left(\frac{\pi}{3600 \cdot 180} \right)^2 \cos 86^\circ \right]$$

$$S_{m36} = \rho_{m36} \Delta \varphi \left[1 + \frac{e^2}{8} \Delta \varphi^2 \cos 2\varphi_M \right] = 30,823 \cdot [1 + 1,38 \cdot 10^{-15}] \Rightarrow \boxed{S_{m36} = 30,823 \text{ m}}$$

$$S_{m43} = \rho_{m43} \Delta \varphi \left[1 + \frac{e^2}{8} \Delta \varphi^2 \cos 2\varphi_M \right] = 30,860 \cdot [1 + 1,38 \cdot 10^{-15}] \Rightarrow \boxed{S_{m43} = 30,860 \text{ m}}$$

EJERCICIO 2.12

$$S_m = \rho_m \Delta \varphi \left[1 + \frac{e^2}{8} \Delta \varphi^2 \cos 2\varphi_M \right]$$

$$e^2 = f(2-f) \Rightarrow e^2 = 0,006694385 \quad ; \quad w_5^3 = (1 - e^2 \operatorname{sen}^2 5^\circ)^{3/2} \Rightarrow w_5^3 = 0,999923723$$

$$\varphi_m = 5^\circ; \quad \Rightarrow \quad \rho_m = \frac{a(1-e^2)}{w_5^3} = \frac{a(1-f(2-f))}{w_5^3} \quad \Rightarrow \quad \rho_m = 6.335.922,575 \text{ m}$$

$$S_m = \rho_m \Delta\varphi \left[1 + \frac{e^2}{8} \Delta\varphi^2 \cos 2\varphi_M \right] = 6.335.922,575 \cdot 10 \frac{\pi}{180} \left[1 + \frac{0,006694385}{8} \left(\frac{10\pi}{180} \right)^2 \cos 10^\circ \right]$$

$S_m = 1.105.854,86 \text{ m}$

EJERCICIO 2.13

$$e^2 = f(2-f) \quad \Rightarrow \quad e^2 = 0,006694385 \quad ; \quad w_{15}^3 = (1 - e^2 \operatorname{sen}^2 15^\circ)^{3/2} \quad \Rightarrow \quad w_{15}^3 = 0,999327417$$

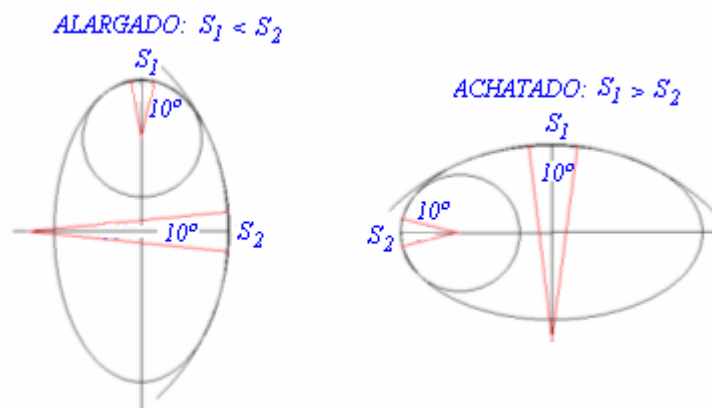
$$\varphi_m = 15^\circ; \quad \Rightarrow \quad \rho_m = \frac{a(1-e^2)}{w_{15}^3} = \frac{a(1-f(2-f))}{w_{15}^3} \quad \Rightarrow \quad \rho_m = 6.339.703,27 \text{ m}$$

$$S_m = \rho_m \Delta\varphi \left[1 + \frac{e^2}{8} \Delta\varphi^2 \cos 2\varphi_M \right] = 6.339.703,27 \cdot 10 \frac{\pi}{180} \left[1 + \frac{0,006694385}{8} \left(\frac{10\pi}{180} \right)^2 \cos 30^\circ \right]$$

$S_m = 1.106.512,29 \text{ m}$

La longitud de este arco es mayor que la del ejercicio anterior, es decir:

$$S_{0^\circ-10^\circ} < S_{10^\circ-20^\circ}$$



Se trata del caso de la derecha de la figura, en el que el arco medido a mayor latitud da mayor longitud, por tanto, se demuestra que en los dos últimos ejercicios se trabaja con un elipsoide achatado por los polos.